CONVECTIVE HEAT EXCHANGE IN FIBROUS MATERIALS AT AN ELEVATED PRESSURE OF THE GASEOUS MEDIUM

G. N. Dul'nev, Yu. P. Zarichnyak,
and A. V. Sharkov

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A procedure is developed for calculating the intensity of heat exchange in porous disordered fibrous materials at an elevated pressure of the gaseous medium. An experimental test is made of the equations obtained.

The main subject of investigation in the present report is convective heat exchange in layers of a uniform, isotropic, permeable, porous material lying between isothermal, impermeable, rigid, plane surfaces having different temperatures. A number of reports [1-11, 15, 16] are devoted to the study of thermal convection in such layers filled with a liquid or gas at normal and elevated pressures. We note that in the majority of the investigations only the large-scale components of the motion of the mobile phase (liquid, gas) are considered, and the linear Darcy law [12, 14]

$$V = -\frac{K_{\text{per}}}{\mu}\operatorname{grad} P \tag{1}$$

is assumed to be valid for their description.

In [1-4] it is assumed that in a porous layer the filtration velocity V and the temperature T can be described by a system of equations of conservation of mass, momentum, and energy the equation of state. This system of equations, together with the conditions at the boundary, is used by the authors to create a simplified theory of convection in a porous medium. To describe the convection processes, they use the dimensionless Grashof and Darcy numbers Gr and Da and the modified Prandtl and Rayleigh numbers Pr* and Ra*:

$$\mathrm{Gr} = \beta g L^3 \Delta T/\nu^2,\ \mathrm{Pr}^* = \nu\rho c_p/\lambda^*,\ \mathrm{Ra}^* = \mathrm{Gr}\,\mathrm{Pr}^*\,\mathrm{Da},\ \mathrm{Da} = K_{\text{per}}/L^2. \tag{2}$$

The intensity of heat transfer through a layer can be represented through the modified average Nusselt number

$$\mathrm{Nu}^* = qL/\lambda^*\Delta T. \tag{3}$$

The thermal conductivity of a porous material in the presence of convection is determined on the basis of the dependence

$$q = \frac{\lambda_{\text{ef}}}{L}\Delta T,$$

and then

$$\mathrm{Nu}^* = \lambda_{\text{ef}}/\lambda^*. \tag{4}$$

Besides the Rayleigh number Ra*, the quantity Nu* within the framework of the formulated problem also depends on the geometry of the layer of porous material: the length H and thickness L of the layer, H/L, and the angle of inclination φ of the force of gravity. The general criterial equation for the mean heat transfer through a layer of porous material has the form

$$\mathrm{Nu}^* = f(\mathrm{Ra}^*,\ H/L,\ \varphi). \tag{5}$$

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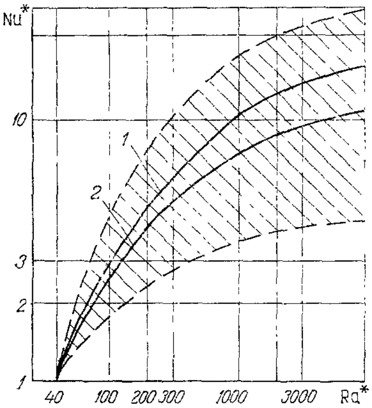

Fig. 1. Heat exchange in horizontal layers of porous material heated from below.

For the case when the cavities between filaments are filled with gas described by the equation of state

$$P = \rho RT, \tag{6}$$

the following expression for the Rayleigh filtration number is given in [3]:

$$\mathrm{Ra}^* = \beta g L \Delta T P^2 c_p K_{\mathrm{per}} / \mu\lambda^* R^2 T^2 = \mathrm{Ra}_0 \mathrm{Da}\, M \overline{P}^2, \tag{7}$$

where Ra_0 is the Rayleigh number calculated at a pressure $P_I = 10^5$ N/m^2; $M = \lambda_g/\lambda^*$; $\overline{P} = P/P_I$ is the pressure ratio.

In [1-4, 7, 8] it is noted that the condition for the occurrence of convection in a layer of permeable porous material heated from is determined by the critical value $\mathrm{Ra}^*_{\mathrm{cr}}$ of the filtration number. For an isotropic porous material

$$\mathrm{Ra}^*_{\mathrm{cr}} = 4\pi^2 \approx 40. \tag{8}$$

A comparison of the criterial equations and experimental results on heat transfer in porous layers obtained by different investigations reveals considerable disagreements in the estimates of the heat-exchange intensity. As an example, in Fig. 1 we present the dependences Nu* = f(Ra*) in horizontal layers of porous material heated from below. The end surfaces of a layer are assumed to be adiabatically insulated. The dashed lines bound the zone of scatter of the experimental data of various authors (from [4]), while curves 1 and 2 give the results of the numerical calculations of [1] and [4], respectively.

As follows from [7], to calculate the intensity of convective heat exchange in a porous layer one must know such parameters as the coefficient of permeability K_{per} and the coefficient of thermal conductivity λ^* of porous materials in the absence of convection. The authors of reports in which the intensity of convective heat exchange in porous layers is studied do not give recommendations on the analytical estimation of these parameters. The latter considerably restricts the possibility of an analytical estimate, since there are test data in the literature on the coefficients K_{per} and λ^* only for a narrow circle of materials, and the experimental means of obtaining them are very laborious. We will use the results of [17-19] for an analytical calculations of these coefficients.

Estimates of the Coefficient of Permeability K_{per}

Different models can lie at the foundation of the equations for calculating K_{per}. The most popular are models consisting of systems of capillarities. The simplest capillary model of the linear type represents a porous medium as a bundle of straight capillaries of constant diameter $\overline{\delta}$ parallel to each other and to the flow. The equation for calculating the coefficient of permeability K_{per} has the form [18]

$$K_{\mathrm{per}} = \Pi \overline{\delta}^2 / 32. \tag{9}$$

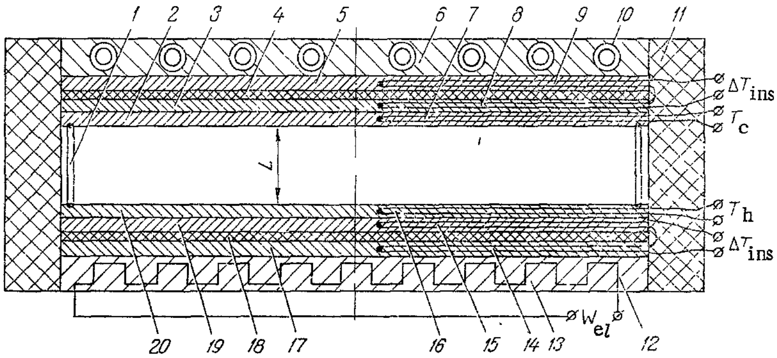

Fig. 2. Diagram of measuring device.

We note that the size $\bar{\delta}$ is indeterminant, while the conduction of the porous medium is analyzed only in one direction. In the so-called parallel type of model it is assumed that the conduction of a porous substance along all three axes is the same; this leads to the dependence [18]

$$K_{per} = \Pi \bar{\delta}^2/96. \tag{10}$$

Equations for models of the "series" type, obtained on the basis of the hydraulic-radius theory and a number of others, are also presented in [18]. To calculate the parameter $\bar{\delta}$ we use the results of [19], where it is shown that the disagreement in the values of the parameter $\bar{\delta}$ based on the data of different authors is especially great for high values of the porosity of the materials. These disagreements can reach hundreds of percent. The most widespread are the functions obtained in [21] and [22] for materials with chaotically distributed fibers:

$$\bar{\delta} = \pi d/8(1-\Pi), \quad \bar{\delta} = \pi d/4(1-\Pi). \tag{11}$$

In subsequent calculations for the determination of $\bar{\delta}$ we will use the arithmetic-mean value obtained from Eqs. (11):

$$\bar{\delta} = \pi d/6(1-\Pi). \tag{12}$$

Persuasive reports on the substantiation of the dependence between porosity and permeability are presently absent. In [18] it is stated that "only by the method of trial and error can one show whether or not a model reflects the characteristic phenomena occurring in a porous medium." We made a comparison between the literature experimental data on the coefficients of permeability of porous materials [6, 17, 18, 20] and the results of calculations by the equations presented in [18]. An analysis of the results obtained showed that for materials having a bulk porosity $\Pi > 0.7$, which are of practical interest, the values of the coefficient K_{per} calculated from Eqs. (9) and (12) are in satisfactory agreement with the measured values.

The effective coefficient of thermal conductivity of fibrous materials in the absence of convection will be calculated by the method developed in [19]. Two groups of equations are offered:

a) for fibrous materials having a chaotic bulk structure

$$\lambda^* = \lambda_1 \left(\frac{0.8}{C^2 + \Phi} + \frac{0.2}{5 \cdot 10^{-3} C^2 + \Phi} \right)^{-1}, \tag{13}$$

where

$$\Phi = \varkappa(1-C)^2 + \frac{2\varkappa C(1-C)}{\varkappa C + 1 - C}; \quad \varkappa = \lambda_2/\lambda_1;$$
$$C = 0.5 \cos \frac{2\Pi - \arccos(2\Pi - 1)}{3}; \tag{14}$$

λ_1 and λ_2 are the thermal conductivities of the material of the fibers and of the gaseous component, respectively, in watts per meter per degree Kelvin;

b) for fibrous materials having an ordered plane structure (mesh)

$$\lambda^* = \lambda_1 \left[(1-\Pi)^2 A + \Pi^2 \varkappa + \frac{4\varkappa \Pi(1-\Pi)}{1-\varkappa} \right], \tag{15}$$

where

$$A = y^2 + \frac{\varkappa(1-y^2)}{1-(1-\sqrt[3]{\varkappa})\sqrt{1-y^2}};\ y = 1{,}13\sqrt{4P_{sp}/E(1-\Pi)^2};$$

P_{sp} is the specific load on the material in newtons per square meter; E is the elastic modulus of the material in newtons per square meter.

Description of Experimental Apparatus

The necessity for a comprehensive experimental test of the proposed model and procedure for calculating the heat-exchange intensity arose in connection with a number of assumptions which were adopted in choosing the model for the calculation of the intensity of heat exchange in porous materials, with the indeterminacy of some of the initial parameters such as the permeability K_{per} and the effective thermal conductivity λ^* of porous materials in the absence of convection, as well as with the very limited experimental data.

The general appearance of the measuring device is shown in Fig. 2. The tests were conducted with two types of cuvettes having heights of 10 and 20 mm. The cuvettes were formed by two duralumin disks 2 and 20 having a diameter of 70 mm. The side walls 1 of the cuvettes were made of plastic 0.2 mm thick. To reduce the radiant heat exchange the inner surface of the components 2 and 20 were chromeplated and polished. The temperature differential between the disks 2 and 20 of the cuvette was produced with the electric heater 12 mounted in the block 13 and the "refrigerator," consisting of a metal disk 6 with tubes 10 soldered into it through which a liquid, preliminarily thermostatically regulated, was pumped. The heat flux through the cuvette was measured by the "auxiliary-wall" method. The calorimeters consisted of ebonite plates 4 and 18 with a thickness of 2 mm and two content copper disks 3, 5 and 17, 19 with a thickness of 3 mm. The components 3, 4, 5 and 17, 18, 19 were cemented together with epoxy resin. Porolon insulation 11 with a thickness of 7 mm was used to reduce the lateral heat exchange between the cuvette and the surrounding medium.

The temperatures of the cuvette walls were measured with Nichrome – Constantan thermocouples 7 and 16 having an electrode diameter of 0.1 mm. The electrical circuit of the apparatus permitted the measurement of the temperature difference between the covers 2 and 20 of the cuvette or the temperature difference of the plates 2 and 20 relative to the temperature of a cold junction located in a massive copper cylinder. The temperature drops in the thermal-insulation layers 4 and 18 of the calorimeters were measured by the 12-junction thermopiles 8, 9 and 14, 15. The temperature drops were recorded by a VK2-20 digital electronic voltmeter.

To investigate the heat-exchange intensity at an elevated pressure of the medium the measuring device was placed in a pressure chamber. The system made it possible to create and measure a pressure of up to $1.6\cdot10^7$ N/m^2. In the range of $1\cdot10^5$–$1\cdot10^6$ N/m^2 the pressure was measured by a standard manometer, while in the range of $1\cdot10^6$–$1.6\cdot10^7$ N/m^2 it was measured by the manometer of accuracy grade 1.5.

Let us consider the working equation for calculating the effective coefficient of thermal conductivity for porous materials at an elevated pressure of the gaseous medium from measurement data. The heat flux Q_{tot} flowing through from the hot wall 20 of the cuvette to the cold wall 2 is equal to the flux Q_{meas} measured by the calorimeter after subtraction of the flux Q_{los} lost through the side surface of the cuvette and the structural elements and thermoelectrodes:

$$Q_{tot} = Q_{meas} - Q_{los}, \tag{16}$$

where

$$Q_{tot} = \frac{\lambda_{ef}}{L}\Delta T S_{I};\ Q_{meas} = \frac{\lambda_{ins}}{\delta_{ins}}\Delta T_{ins} S_{II}. \tag{17}$$

Substituting the expression for the heat fluxes into Eq. (16) and taking $S_I = S_{II} = S_c$, we find the effective coefficient of heat conduction between the cuvette walls:

$$\lambda_{ef} = \frac{\lambda_{ins} L \Delta T_{ins}}{\delta_{ins}\Delta T} - \frac{Q_{los} L}{S_K \Delta T} = AL\frac{\Delta T_{ins}}{\Delta T} - B\frac{L}{\Delta T}. \tag{18}$$

To determine the coefficients A and B we made a series of calibration tests on cuvettes filled with argon gas and on specimens of plastic 18 and 28 mm thick in the pressure range of $(1\text{–}120)\cdot10^5$ N/m^2. It was found that the value of the coefficient A is 7.0 W/m^2. A series of graphs was plotted to determine B, since this coefficient is a function of the pressure of the gaseous medium and of the temperature drop between the cuvette and the surrounding medium. Depending on these parameters, the coefficient B varied in the range of 50–185 W/m^2.

TABLE 1. Characteristics of Fibrous Materials Investigated

No. of points in Fig. 3	Material	$d \cdot 10^6$, m	λ_1, W/m·°K	Gas filling	Π	λ^*, W/m·°K
1	Cotton	30	0,50	Argon	0,95	0,043
2	Cotton	30	0,50	»	0,97	0,039
3	Cotton	30	0,50	»	0,99	0,040
4	Wool	60	0,30	»	0,95	0,040
5	Wool	60	0,30	»	0,97	0,040
6	Wool	60	0,30	»	0,99	0,043
7	Dacron	30	0,22	»	0,95	0,039
8	Dacron	30	0,22	»	0,98	0,040
9	Orlon	30	0,20	»	0,95	0,049
10	Orlon	30	0,20	»	0,98	0,550
11	Tangled copper	150	390	»	0,95	0,087
12	Tangled copper	150	390	»	0,97	0,082
13*	Fiber glass	20	1,0	Air	0,95	0,048

*Test data of [9].

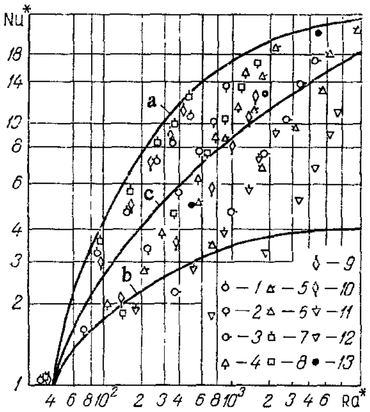

Fig. 3. Results of an experimental investigation of heat-exchange intensity in disordered, isotropic, highly porous materials, the characteristics of which are given in Table 1.

We note that the remaining parameters were varied in the following limits: $10°K \leq \Delta T \leq 30°K$; $25°K \leq \Delta T_{ins} \leq 45°K$; $0.04\ W/m \cdot °K \leq \lambda_{ef} \leq 0.82\ W/m \cdot °K$.

A calculation of the measurement error showed that the absolute error in measurements of the effective coefficient of thermal conductivity does not exceed $\Delta\lambda_{ef} \leq 0.014\ W/m \cdot °K$ in the most unfavorable case.

Results of the Experimental Investigation and Their Generalization

To run the tests we used fibrous materials for which data on the properties are given in Table 1. The porosity of the materials varied in the range of 0.95–0.99. Argon at a pressure $P = (1\text{-}120) \cdot 10^5\ N/m^2$ was used as the gaseous medium.

The data of our tests are represented by points in Fig. 3. In the same graph the curves a and b the scatter field of the results of various authors according to the data of [4] while the curve c approximates our experimental data.

On the basis of the treatment and approximation of the measurement results (curve c), the following functions are suggested for calculating the heat-exchange intensity in horizontal layers of fibrous materials:

$$\text{for} \quad Ra^* \leqslant 40 \quad Nu^* = 1;$$

$$\text{for } 40 < Ra^* < 400 \quad Nu^* = 0.4\,(Ra^*)^{0.5} - 1.5;$$
$$\text{for } 400 \leqslant Ra^* < 10000 \quad Nu^* = 0.17\,(Ra^*)^{0.5} + 2.8. \qquad (19)$$

The following procedure for calculating the heat-exchange intensity in fibrous materials is proposed on the basis of an analysis of the literature data and the experimental investigation which was performed.

The initial information for the calculation of Nu* are: the diameter d and thermal conductivity λ_1 of the fibers, the porosity Π of the material, the thermal conductivity λ_2 of the gas filling, the thickness L of the interlayer, and the temperatures T_1 and T_2 of the surfaces bounding it.

Using thermophysical data on the gas filling, the dimensions of the cavity, and the temperatures at its boundaries the value of the criterion Ra_0 is calculated from Eq. (2), in which all the parameters are determined at a pressure $P = 1 \cdot 10^5$ N/m^2 and a temperature T_m equal to the arithmetic-mean value of the wall temperatures of the layer.

The Darcy number Da is determined by Eqs. (2), (9), and (12) from the known values of the porosity Π of the fibrous material and the fiber diameter d.

The parameter M is calculated from (13)-(15) and the modified Rayleigh number Ra* from (7).

The average intensity of heat transfer Nu* through the layer is determined from Eqs. (19).

The standard deviation of the calculated results from the experimental data in the range of $1 < Ra^* < 10^4$ does not exceed 25% at a confidence level of 0.67.

The results of the investigation presented explain the considerable increase in the effective thermal conductivity of interlayers filled with porous material (up to dozens of times upon a pressure rise only up to $1.5 \cdot 10^7$ N/m^2). The latter confirms the necessity of allowing for the convective mechanism of heat transfer in technological processes, power installations, and various kinds of instruments and devices operating at increased pressures of the working substance (gas).

NOTATION

λ_{ef}	is the effective thermal conductivity of layer of porous material, W/m·°K;
λ^*	is the same in the absence of convertion, W/m·°K;
λ_g	is the thermal conductivity of gas filling, W/m·°K;
ν	is the coefficient of kinematic viscosity, m^2/sec;
μ	is the coefficient of dynamic viscosity, N·sec/m^2;
ρ	is the density, kg/m^3;
K_{per}	is the coefficient of permeability, m^2;
L	is the height of test layer, m;
ΔT	is the temperature drop in test layer, °K;
T	is the temperature, °K;
P	is the pressure of filling medium, N/m^2;
β	is the coefficient of volumetric expansion of gas, 1/°K;
g	is the free-fall acceleration, m/sec^2;
Π	is the porosity of material in fractions of a unit;
R	is the gas constant, J/°K;
Q	is the heat flux, W;
λ_{ins}	is the thermal conductivity of insulating interlayer of calorimeter, W/m·°K;
δ_{ins}	is the thickness of insulating interlayer of calorimeter, m;
ΔT_{ins}	is the temperature drop in thermally insulating interlayer of calorimeter, °K;
c_p	is the specific heat of gas filling at constant pressure, J/kg·°K;
$\bar{V}$	is the filtration velocity, m/sec;
$\bar{\delta}$	is the mean distance between fibers, m;
d	is the fiber diameter, m;

the index 0 means that the thermophysical parameters are calculated at a pressure $P = 1 \cdot 10^5$ N/m^2.

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